Final Exam
Cumulative

Answer the following questions. *Answers without proper evidence of knowledge will not be given credit.* Make sure to make reasonable simplifications. Do not approximate answers. Give exact answers. **No calculators are allowed on this exam.** Unless otherwise stated, every problem is worth 10 points (making a total of 150 points).

True or False (1 point each)

1. _____ Any polynomial can be graphed as a transformation of the graph of $y = x^2$.
2. _____ For any angle $\alpha \in \mathbb{R}$ we have that $(\sin(\alpha) + \cos(\alpha))^2 = 1$.
3. _____ Given any one-to-one function $f(x)$, we can define an inverse function $f^{-1}(x)$ such that $f^{-1}(f(x)) = x$ for every x in the domain of $f(x)$.
4. _____ If $a > 1$ then the function $f(x) = a^x$ has a horizontal asymptote on the x-axis.
5. _____ For any value of r, $\frac{1}{1-r} = 1 + r + r^2 + r^3 + \cdots$.
6. _____ If you know any two angles and one side of a triangle, then there is only one possibility for the other sides and angle of the triangle.
7. _____ If $p(x)$ and $q(x)$ are polynomials and $\frac{p(x)}{q(x)}$ is a rational expression in lowest terms, then there is a vertical asymptote at all the zeros of $q(x)$.
8. _____ Any polynomial $p(x) = x^n + a_{n-1}x^{n-1} + \cdots + a_1x + a_0$ of degree n has exactly n roots in the complex numbers, counting multiplicity.
9. _____ $\frac{1}{x+1} + \frac{1}{x-1} = \frac{1}{x}$
10. _____ For any triangle, $\tan\alpha = \frac{opposite}{adjacent}$.

1. Match each function with its appropriate graph. Note that there are more graphs than functions.

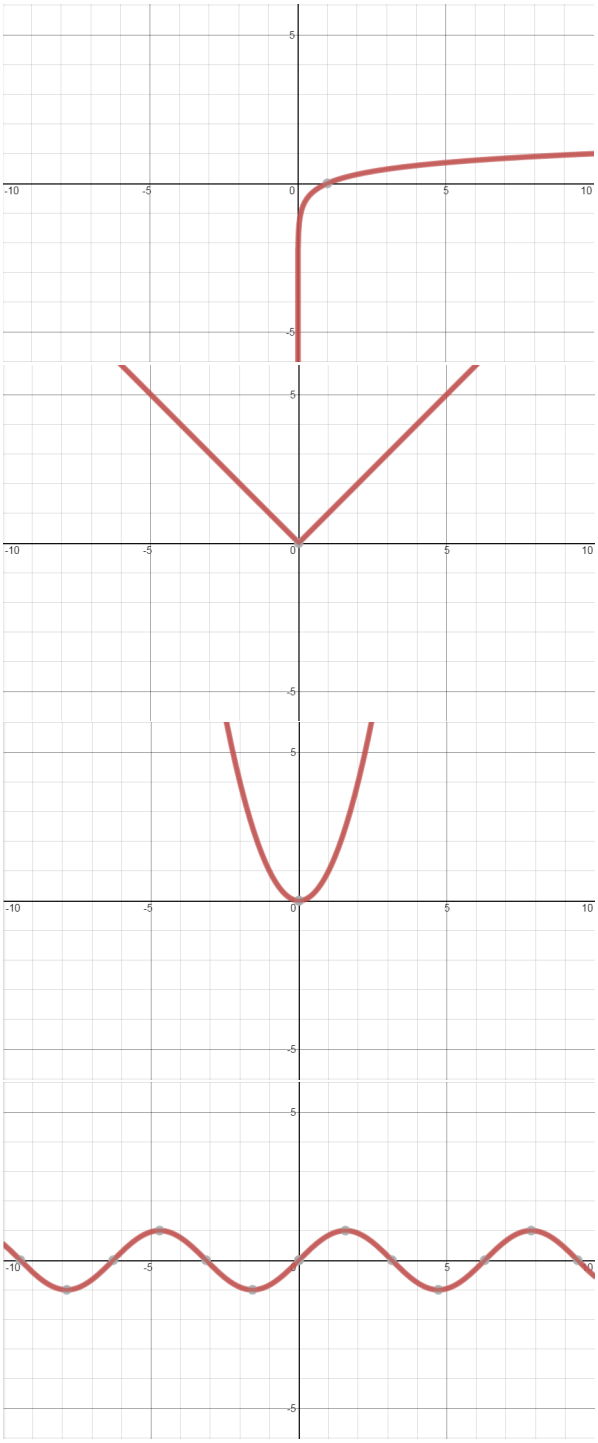
a. ____ $f(x) = \sin(x)$

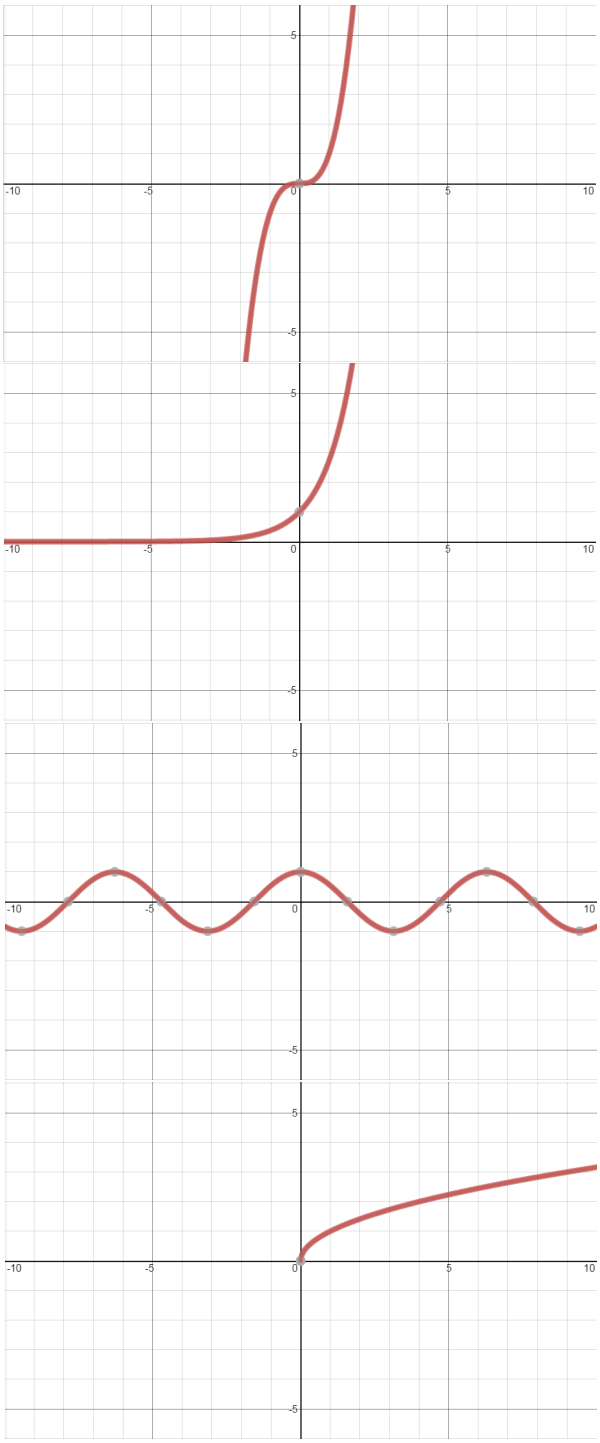
b. ____ $g(x) = e^x$

c. ____ $h(x) = x^2$

d. ____ $p(x) = \log(x)$

e. ____ $q(x) = \sqrt{x}$





2. The function $f(x)$ is pictured to the right. Match each transformation with its appropriate graph.

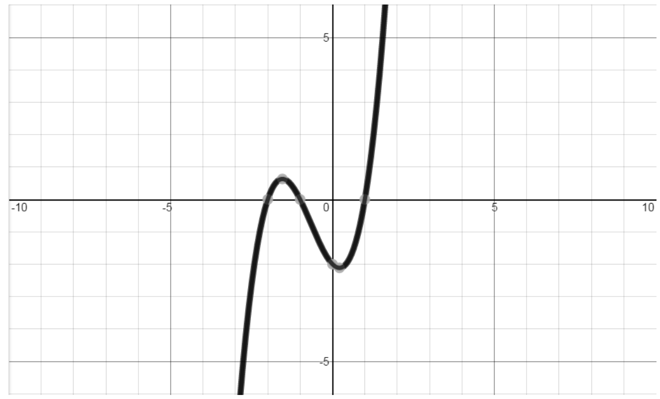
a. ____ $2f(x)$

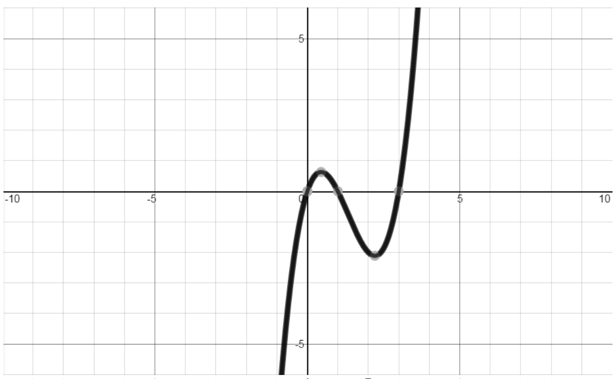
b. ____ $f(x) + 3$

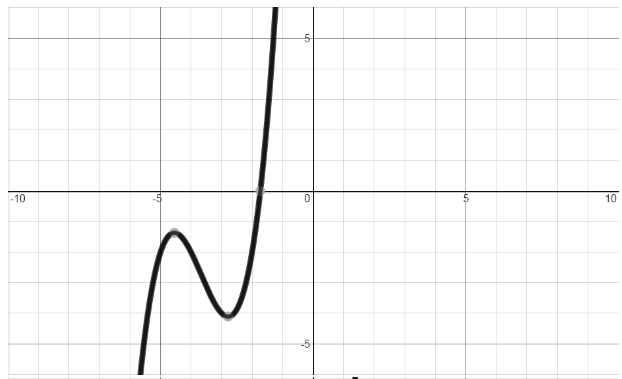
c. ____ $f(x - 2$

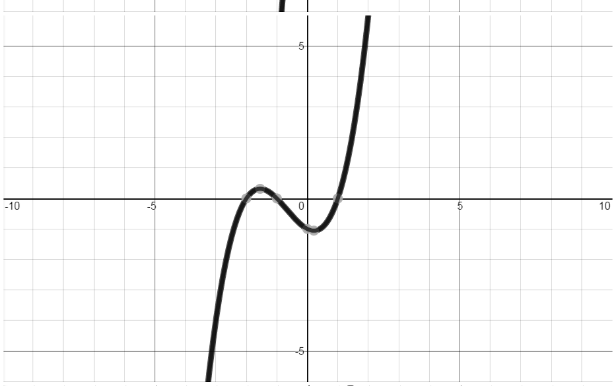
d. ____ $\frac{1}{2}f(x)$

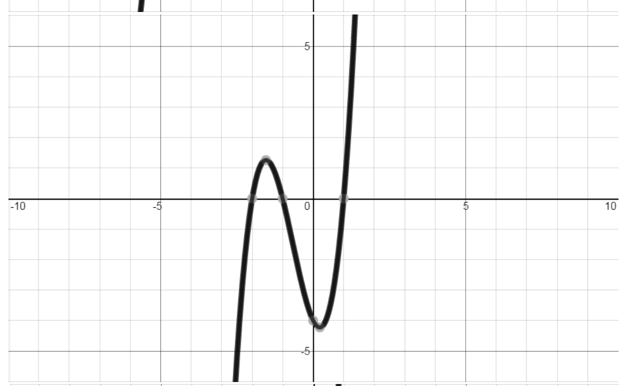
e. ____ $f(x + 3) - 2$

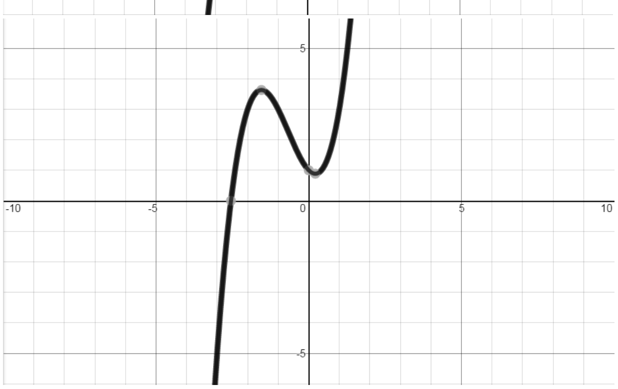


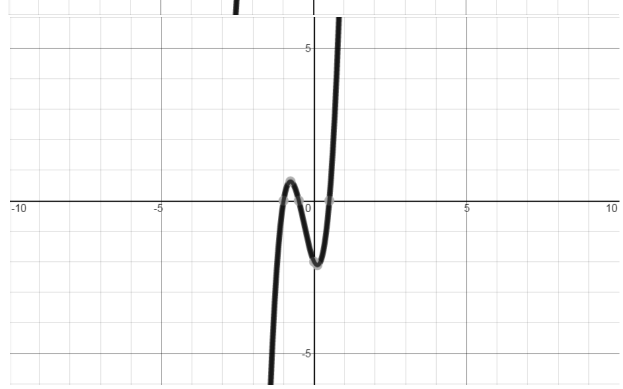




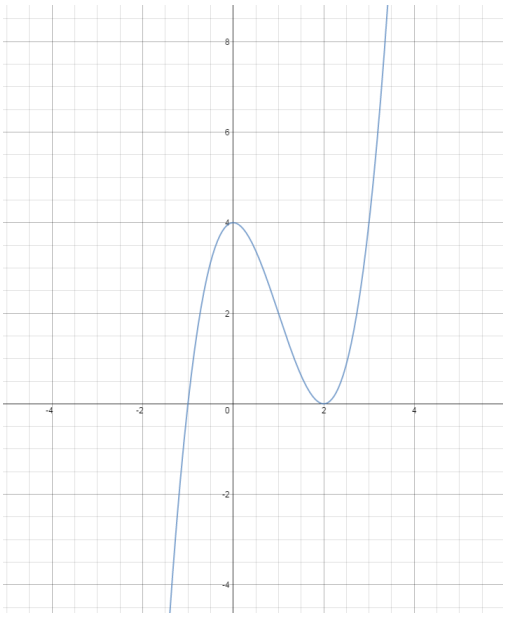








3. Consider the functions f and g given below.



$g(x)$ contains the orderered pairs

$$\{(1,3),(2,5),(0,-1),(4,1),(3,2)\}$$

a) Is the function $f(x)$ invertible?

b) Find $f(g(3))$.

c) Find $g(f(1))$.

4. Given the function $y = 2x^2 - 12x + 22$, by Completing the square write it in vertex form. Also State the vertex.

5. Solve for x in the equation $\log_2(x^3) + \log_2((x-2)^3) = 9$.

6. Verify the following trigonometric identity.

$$\frac{\sin x}{1-\sec x} - \frac{\sin x}{1+\sec x} = -2\cot x$$

7. Consider the following functions $f(x) = x^2 - 1$, $g(x) = \cos(x)$, $h(x) = \arcsin(x)$, $k(x) = \sqrt{3x}$, and $j(x) = e^x$

a) Find $f \circ g(x)$

b) Find $j \circ f \circ g(\pi)$

c) Find $k \circ f(x)$

8. Let α be an angle in Quadrant I and β be an angle in Quadrant II. Also assume $\sin\alpha = 3/5$ and $\cos\beta = -2/5$. Find $\cos(\alpha + \beta)$.

9. Consider the sequence given by $a_n = \frac{(n+3)!}{n!}$.

a) Find the first 4 terms of the sequence.

b) Rewrite the general term a_n as a product without the factorials.

10. Recall that the difference quotient

$$\frac{f(x+h)-f(x)}{h}$$

gives the average rate of change between two x-values for the function $f(x)$. Evaluate and simplify the difference quotient for

$$f(x) = 2x^2 - 3x + 5$$

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11. For each of the following, solve for the indicated variable.

a) Solve for x. $\log(x) = y$

b) Solve for w. $\cos(w) = z$

12. Consider the function $f(x) = \frac{\sqrt{3x+7}}{2}$.

a) Find the domain and range of $f(x)$.

b) Find the inverse function of $f(x)$.

c) Find the domain and range of $f^{-1}(x)$. (**Hint:** How are the domain and range of inverse functions related?)

13. Bilbo averaged 24 miles per day (mpd) on the trip from Hobbiton to Erebor on an adventure. On the return trip he averaged 18 mpd and the trip took 3 days longer. How far is it from Hobbiton to Erebor?

There			
Back Again			

14. In the first section we covered this semester we said that the set of rational numbers are all numbers that have a terminating or repeating decimal expansion. Write the decimal expansion below as a rational number (i.e. one integer divided by another):

.181818181818...

Extra Credit:

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1. Write down one thing that you learned this semester. (5 points)

2. Solve the triangle with sides $a = \sqrt{3} + 1$, $b = 1$, and $c = \frac{3\sqrt{2}+\sqrt{6}}{2}$. (10 points)